
ADDITIONAL MATHEMATICS

4037/21

Paper 2

May/June 2017

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

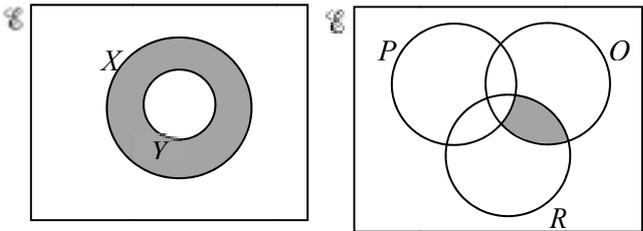
- M Method marks, awarded for a valid method applied to the problem.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘**dep**’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

| | |
|------|----------------------------|
| awrt | answers which round to |
| cao | correct answer only |
| dep | dependent |
| FT | follow through after error |
| isw | ignore subsequent working |
| nfww | not from wrong working |
| oe | or equivalent |
| rot | rounded or truncated |
| SC | Special Case |
| soi | seen or implied |

| Question | Answer | Marks | Guidance |
|----------|--|------------|---|
| 1 | Integrates | M1 | must be clear attempt to integrate at least one term |
| | $[y =]x^4 + x (+c)$ | A1 | Both terms correct |
| | $17 = 2^4 + 2 + c$ | DM1 | Substitution of $x = 2, y = 17$ to find c |
| | $y = x^4 + x - 1$ cao | A1 | must have $y =$ |
| 2(a) | $2\sqrt{6} \times 3\sqrt{3} = 6\sqrt{18}$ oe | M1 | method must be shown – simplifies and combines product |
| | $18\sqrt{2}$ | A1 | If all over common denominator then consider the product for M1A1 |
| | $9\sqrt{2}$ oe soi leading to final answer of $27\sqrt{2}$ | B1 | |
| 2(b) | $[x =] \frac{6 + \sqrt{3}}{2 - \sqrt{3}}$ | M1 | Expanding and making x subject – condone slips but must be of equivalent difficulty |
| | $[x =] \frac{6 + \sqrt{3}}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$ oe and multiplies out numerator and denominator | M1 | numerator at least 3 terms; $12 + 2\sqrt{3} + 6\sqrt{3} + 3$ |
| | $15 + 8\sqrt{3}$ | A1 | |
| 3(i) | $\frac{2x}{x^2 + 1}$ final answer | B2 | B1 for $\frac{1}{x^2 + 1} \times (ax + b)$, a or b must be non-zero |
| 3(ii) | $\delta y = \text{their} \left(\frac{2(3)}{(3)^2 + 1} \right) \times h$ or better | M1 | Substitutes $x = 3$ into <i>their</i> $\frac{dy}{dx}$ and multiplies by h |
| | $\frac{6}{10}h$ oe | A1 | |
| 4(a)(i) | 36 | B1 | |
| 4(a)(ii) | 7 | B1 | |
| 4(b) | $[y =] 5 \sin 4x + 7$ | B4 | B1 for each of 5, 4 and 7 and B1 for sine Accept $a = 5, b = 4, c = 7$ for B3 |

| Question | Answer | Marks | Guidance |
|----------|--|-----------|---|
| 5(i) | $16 + 32ax + 24a^2x^2 + 8a^3x^3 + a^4x^4$ | B2 | B1 for at most 2 terms incorrect or missing or for correct but unsimplified form SC1 for $16 + 32ax + 24ax^2 + 8ax^3 + ax^4$ or all terms correct listed |
| 5(ii) | $24a^2 = 8a^3$ and solves to given answer | B1 | or verifies that $a = 3$ leads to coeff of 216 for both terms must be from correct terms in (i) |
| 5(iii) | $x = -0.01$ or $ax = -0.03$ soi | M1 | |
| | $16 + 32(3)(-0.01) + 24(9)(-0.01)^2$ leading to $16 - 0.96 + 0.0216$ or $15.06\dots$ isw | A1 | Must show clear substitution into their expansion for A1 and reach a value which rounds to 15.1 |
| 6(i) | $(\mathbf{M} =) \begin{pmatrix} 90 & 10 & 30 \\ 0 & 45 & 0 \\ 25 & 0 & 15 \\ 10 & 0 & 100 \end{pmatrix}$ | B1 | columns and/or rows may be interchanged but must be consistent |
| 6(ii) | $(\mathbf{LM} =) (1 \ 1 \ 1 \ 1) \begin{pmatrix} 90 & 10 & 30 \\ 0 & 45 & 0 \\ 25 & 0 & 15 \\ 10 & 0 & 100 \end{pmatrix} = (125 \ 55 \ 145)$ | B1 | Answer must be of correct order and must be consistent with a correct M |
| 6(iii) | The total numbers of each type of ticket sold by all 4 cinemas oe | B1 | |
| 6(iv) | $(\mathbf{N} =) \begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix}$ | B1 | Calculation not required |
| | The total income of all (4) cinemas or other valid comment e.g. total income from all ticket sales | B1 | Total cost/value of tickets etc. |
| 7(a) |  | B2 | B1 for each |
| 7(b)(i) | $n(M \cap D) = 0$ or $M \cap D = \emptyset$ | B1 | No additional brackets e.g. $M \cap D = \{\emptyset\}$ is B0 |

| Question | Answer | Marks | Guidance |
|----------|--|-----------|--|
| 7(b)(ii) | | B3 | <p>B1 correct intersection of circles with 12 and 25 correct</p> <p>B1 33, 2, 11 correctly placed</p> <p>B1FT 17; must be on the Venn diagram and identified as the required answer</p> <p>FT on 100– (sum of <i>their</i> 5 correctly positioned values)</p> |
| 8(a) | ${}^{30}P_2 = 870$ | B1 | |
| 8(b)(i) | ${}^2C_1 \times {}^{14}C_{10}$ oe (2×1001) | M1 | Condone $\binom{14}{4}$ for $\binom{14}{10}$ |
| | 2002 | A1 | implies M1 |
| 8(b)(ii) | $({}^2C_1 \times {}^5C_4 \times {}^9C_6) + ({}^2C_1 \times {}^5C_5 \times {}^9C_5)$ oe $(840 + 252)$ ${}^2C_1 \times {}^{14}C_{10}$ – or $({}^2C_1 \times {}^5C_1 \times {}^9C_9 + {}^2C_1 \times {}^5C_2 \times {}^9C_8 + {}^2C_1 \times {}^5C_3 \times {}^9C_7)$ $\{2002 - (10 + 80 + 720)\}$ | M3 | <p>M3 for fully correct method soi</p> <p>M2 for all necessary products but not summed with no extra products seen soi</p> <p>M1 for one correct three term product soi</p> |
| | 1092 | A1 | implies M3 |
| 9(i) | Substitution of $y = 2(1 - x)$ | M1 | Must be attempt at full substitution. Condone one sign error in substitution. Condone omission of = 0 or incorrect rhs |
| | $-3x^2 + 2x + 1 = 0$ oe $(3x^2 - 2x - 1 = 0)$ | A1 | Terms collected |
| | Solving <i>their</i> quadratic found from eliminating one variable $(3x + 1)(1 - x)$ or $(3x + 1)(x - 1)$ | M1 | can be implied by a correct pair of x values |
| | $\left(-\frac{1}{3}, \frac{8}{3}\right)$ oe and $(1, 0)$ oe isw nfw | A2 | A1 for each or A1 for a correct pair of x -coordinates or a correct pair of y -coordinates |

| Question | Answer | Marks | Guidance | | | | | | | | | | |
|---|--|--|---|--------|-----|-----|---------|------|------|------|--------|-----------|--|
| 9(ii) | $[m =] \frac{1}{2}$ cao | B1 | | | | | | | | | | | |
| | $\left(\frac{1}{3}, \frac{4}{3}\right)$ | B1 | FT | | | | | | | | | | |
| | $y - \text{their} \frac{4}{3} = \text{their} \frac{1}{2} \left(x - \text{their} \frac{1}{3}\right)$ | M1 | or $y = \text{their} \frac{1}{2}x + c$ and substitutes their midpoint and reaches $c = \dots$ | | | | | | | | | | |
| | $6y - 3x = 7$ | A1 | allow any equivalent form with integer coeffs/constant | | | | | | | | | | |
| 10(i) | <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>t</td> <td>1</td> <td>1.5</td> <td>2</td> <td>2.5</td> </tr> <tr> <td>$\ln P$</td> <td>1.48</td> <td>2.12</td> <td>2.76</td> <td>3.4(0)</td> </tr> </table> | t | 1 | 1.5 | 2 | 2.5 | $\ln P$ | 1.48 | 2.12 | 2.76 | 3.4(0) | M1 | allow $\ln P$ values to 1 dp rounded or truncated (1.5, 2.1, 2.8, 3.4) |
| | t | 1 | 1.5 | 2 | 2.5 | | | | | | | | |
| $\ln P$ | 1.48 | 2.12 | 2.76 | 3.4(0) | | | | | | | | | |
| single ruled line drawn within tolerance at least for t between 1 and 2.5 | A1 | All points within 1 square of line / must not pass through origin | | | | | | | | | | | |
| 10(ii) | $e^{\text{their}3}$ | M1 | | | | | | | | | | | |
| | 18 to 22.2 | A1 | | | | | | | | | | | |
| 10(iii) | $(0, c)$ with $0.1 \leq c \leq 0.3$ (0.2) | B1 | allow $y = c$ condone $c = \dots$ | | | | | | | | | | |
| | m in the range $1.25 \leq m \leq 1.34$ (1.28) | B1 | | | | | | | | | | | |
| 10(iv) | $\ln P = (\text{their}1.28)t + \text{their}0.2$ | M1 | or $\ln P = (\ln b)t + \ln a$ | | | | | | | | | | |
| | $P = e^{(\text{their}1.28)t + \text{their}0.2}$ | M1 | or $\ln b = m = \text{their}1.28$ and $\ln a = c = \text{their}0.2$ | | | | | | | | | | |
| | $P = e^{\text{their}0.2} e^{(\text{their}1.28)t}$ | A1 | or $1.10 \leq a \leq 1.35$ $3.49 \leq b \leq 3.82$ | | | | | | | | | | |
| 10(v) | $1000 * e^{\text{their}0.2} \times e^{\text{their}1.28t}$ or $1000 * \text{their} a \times \text{their} b^t$ | M1 | A correct relationship e.g. $1.3t * \ln(1000) - 0.2$ where * is = or an inequality sign | | | | | | | | | | |
| | 5.3 | A1 | 5.2 to 5.5 must be to 1dp | | | | | | | | | | |

| Question | Answer | Marks | Guidance |
|----------|--|------------|---|
| 11(i) | $\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} = \frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$ oe | B2 | B1 for either $\cot x = \frac{\cos x}{\sin x}$ or $\tan x = \frac{\sin x}{\cos x}$ used B1 for correctly placing over a common denominator or for splitting into 3 correct terms not just for stating or working from both sides |
| | Valid use of Pythagorean identity e.g. $\cos^2 x + \sin^2 x = 1$ | B1 | |
| | Simplification to $\sec x$ (correct solution only) | B1 | not if working from both sides |
| 11(ii) | $\cos x = \frac{1}{2}$ soi | M1 | |
| | 60, 300 | A1 | Correct pair |
| | $\cos x = -\frac{1}{2}$ soi | M1 | |
| | 120, 240 | A1 | Correct pair |
| 12(i) | $\left[v = \frac{d(3t - \cos 5t + 1)}{dt} \right] = 3 + 5 \sin 5t$ | B2 | B1 for either with no other terms or for both with 1 extra |
| | <i>their</i> $(3 + 5 \sin 5t) = 0$ | M1 | Must be from an attempt to differentiate |
| | awrt 0.76 | A1 | 0.7570187525 |
| | awrt 1.13 | A1 | 1.12793684 |
| | substitutes <i>their</i> t values into s (4.07..., 3.58...) | DM1 | must be two values |
| | 0.48 to 0.49 [m] | A1 | Final A1 may imply earlier A1 s |
| 12(ii) | $25 \cos 5t$ | M1 | Differentiating <i>their</i> v correctly providing at least 2 terms with one trig function |
| | -25 | A1 | Ignore +25 following -25 |